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The Existence of Multiple Equilibria in the UPFC Power Injection Model

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The Existence of Multiple Equilibria in the UPFC Power Injection Model

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***Abstract*—This letter shows the existence of multiple equilibria that arise from the use of the state model of the unified power flow controller (UPFC). These multiple equilibria can arise from a common power injection model for the same terminal conditions of shunt bus voltage and series active and reactive power injections. The multiple equilibria result in two or more sets of eigenvalues, some of which may indicate an unstable operating condition. Therefore, the use of the UPFC power injection model must be used with caution to ensure stable operation of the UPFC.**

***Index Terms*—Oscillation damping, power system stability, unified power flow controller (UPFC).**

I. Introduction

The unified power flow controller (UPFC) power injection model is widely used for power system simulations (recent examples include [1]–[3]). In the power injection model, the impact of the UPFC in the network is represented by its series and shunt current injections, or similarly, its series and shunt active and reactive power injections. A common approach to incorporating the UPFC power injection model into the system is to represent the UPFC as two buses: a "PQ" bus at the receiving end in which both active and reactive power are specified, and a "PV" bus at the sending end in which voltage and active power are specified [4]. In this letter, it will be shown that if the power injection model is used instead of the dynamic model for the same operating conditions, then multiple equilibria (with possibly different stability properties) can exist.

II. UPFC State Model

The UPFC is a combination of the static synchronous compensator (STATCOM) and static series synchronous compensator (SSSC) as shown in Fig. 1. The series connected inverter injects a voltage with controllable magnitude and phase angle in series with the transmission line, thereby providing active and reactive power to the transmission line. The shunt-connected inverter provides the active power drawn by the series branch and the losses and can independently provide reactive compensation to the system. The UPFC state model is

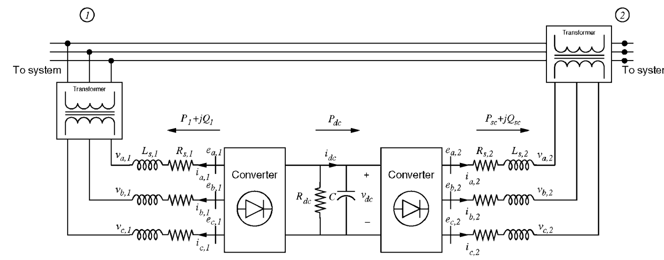

Fig. 1. Unified power flow controller diagram.

$$\frac{1}{\omega_s}\frac{d}{dt}i_{d_1} = \frac{k_1 V_{dc}}{L_{s_1}}\cos(\alpha_1+\theta_1) + \frac{\omega}{\omega_s}i_{q_1} - \frac{R_{s_1}}{L_{s_1}}i_{d_1} - \frac{V_1}{L_{s_1}}\cos\theta_1 \quad (1)$$

$$\frac{1}{\omega_s}\frac{d}{dt}i_{q_1} = \frac{k_1 V_{dc}}{L_{s_1}}\sin(\alpha_1+\theta_1) - \frac{R_{s_1}}{L_{s_1}}i_{q_1} - \frac{\omega}{\omega_s}i_{d_1} - \frac{V_1}{L_{s_1}}\sin\theta_1 \quad (2)$$

$$\frac{1}{\omega_s}\frac{d}{dt}i_{d_2} = -\frac{R_{s_2}}{L_{s_2}}i_{d_2} + \frac{\omega}{\omega_s}i_{q_2} + \frac{k_2}{L_{s_2}}\cos(\alpha_2+\theta_1)V_{dc} - \frac{1}{L_{s_2}}(V_2\cos\theta_2 - V_1\cos\theta_1) \quad (3)$$

$$\frac{1}{\omega_s}\frac{d}{dt}i_{q_2} = -\frac{R_{s_2}}{L_{s_2}}i_{q_2} - \frac{\omega}{\omega_s}i_{d_2} + \frac{k_2}{L_{s_2}}\sin(\alpha_2+\theta_1)V_{dc} - \frac{1}{L_{s_2}}(V_2\sin\theta_2 - V_1\sin\theta_2) \quad (4)$$

$$\frac{C}{\omega_s}\frac{d}{dt}V_{dc} = -k_1\cos(\alpha_1+\theta_1)i_{d_1} - k_1\sin(\alpha_1+\theta_1)i_{q_1} - k_2\cos(\alpha_2+\theta_1)i_{d_2} - k_2\sin(\alpha_2+\theta_1)i_{q_2} - \frac{V_{dc}}{R_{dc}} \quad (5)$$

where the parameters are as in [5]. The currents i_{d1} and i_{q1} are the dq components of the shunt current. The currents i_{d2} and i_{q2} are the dq components of the series current. The voltages $V_1\angle\theta_1$ and $V_2\angle\theta_2$ are the shunt and series voltage magnitudes and angles, respectively. V_{dc} is the voltage across the dc capacitor, R_{dc} represents the switching losses, R_{s_1} and L_{s_1} are the shunt transformer resistance and inductance, respectively, and R_{s_2} and L_{s_2} are the series transformer resistance and inductance, respectively. The control parameters $k_1(k_2)$ and $\alpha_1(\alpha_2)$ are, respectively, the modulation gain and voltage phase angle of the shunt (series) injected voltage.

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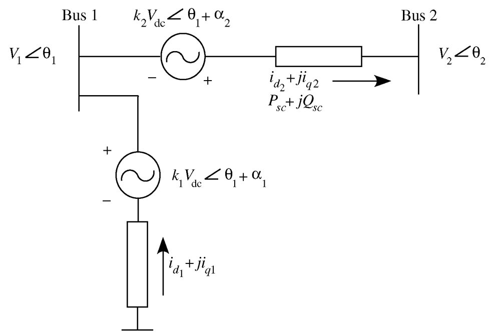

Fig. 2. UPFC equivalent model.

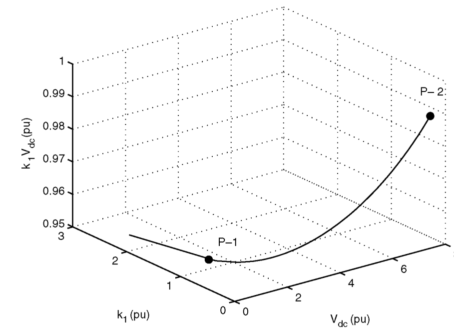

Fig. 3. UPFC parameters for variations in P_1.

The power balance equations at bus 1 (sending) are

$$0 = V_1\left((i_{d_1} - i_{d_2})\cos\theta_1 + (i_{q_1} - i_{q_2})\sin\theta_1\right) - V_1\sum_{j=1}^{n} V_jY_{1j}\cos(\theta_1 - \theta_j - \phi_{1j}) \quad (6)$$

$$0 = V_1\left((i_{d_1} - i_{d_2})\sin\theta_1 - (i_{q_1} - i_{q_2})\cos\theta_1\right) - V_1\sum_{j=1}^{n} V_jY_{1j}\sin(\theta_1 - \theta_j - \phi_{1j}) \quad (7)$$

and at bus 2 (receiving)

$$0 = V_2\left(i_{d_2}\cos\theta_2 + i_{q_2}\sin\theta_2\right) - V_2\sum_{j=1}^{n} V_jY_{2j}\cos(\theta_2 - \theta_j - \phi_{2j}) \quad (8)$$

$$0 = V_2\left(i_{d_2}\sin\theta_2 - i_{q_2}\cos\theta_2\right) - V_2\sum_{j=1}^{n} V_jY_{2j}\sin(\theta_2 - \theta_j - \phi_{2j}). \quad (9)$$

Fig. 2 shows a power injection model of the UPFC. The series branch shows the series injected voltage (controllable by varying k_2 and α_2) and the shunt branch with voltage controlled by k_1 and α_1.

Combining (1)–(9) yields nine equations with thirteen unknowns; therefore, additional constraints are necessary to fully determine the operating equilibrium.

In the power injection model, three parameters may be arbitrarily set: the shunt bus voltage magnitude and the series active and reactive powers such that

$$V_{sc} = V_1 \quad (10)$$

$$P_{sc} = V_{d_2}i_{d_2} + V_{q_2}i_{q_2} \quad (11)$$

$$Q_{sc} = V_{q_2}i_{d_2} - V_{d_2}i_{q_2} \quad (12)$$

where V_{sc}, P_{sc}, and Q_{sc} are the specified desired values.

Since the power injection model is lossless, the shunt power P_1 is typically set to P_{sc} as well (being a "PV" bus). However, in the state model, the shunt power must account for losses in the converter such that

$$-P_1 = P_{sc} + P_{loss} = P_{sc} + R_{s_1}\left(i_{d_1}^2 + i_{q_1}^2\right) + R_{s_2}\left(i_{d_2}^2 + i_{q_2}^2\right) + \frac{1}{R_{dc}}V_{dc}^2 \quad (13)$$

thus providing the thirteenth equation. Therefore, for the same specified values of V_{sc}, P_{sc}, and Q_{sc}, multiple solutions for the remaining variables may exist depending on the choice of P_1. The power injection model in which $P_1 = P_{sc}$ is just one of many solutions that exist to the model of (1)–(12).

In applications in which a dynamic model is used, typically the dc link voltage (V_{dc}) is controlled. By controlling V_{dc}, the user is indirectly specifying the value of P_1 since the shunt active power is used to maintain V_{dc}. However, the power injection model is independent of the value of V_{dc}; therefore, the value of P_1 can be arbitrarily chosen, which may lead to inconclusive results concerning stability.

III. Illustrative Example

In this example, a UPFC is placed in the IEEE 118-bus system with the following parameters (in per unit):

$$R_{s_1} = 0.01 \quad \omega_s L_{s_1} = 0.15 \quad R_{dc} = 100$$
$$R_{s_2} = 0.001 \quad \omega_s L_{s_2} = 0.015 \quad C = 1.1364.$$

Fig. 3 shows the variation in k_1 and V_{dc} as P_1 is varied and P_{sc}, Q_{sc}, and V_{sc} are held constant. Note that the product k_1V_{dc} remains nearly constant; thus, the magnitude of the injected voltage remains near 1.0 with only a few percent variation to regulate the shunt voltage at the desired V_{sc}.

Consider the two points (P-1 and P-2) indicated in Fig. 3. These two points correspond to the same operating conditions where

$$P_{sc} = -0.1178 \text{ p.u.} \quad Q_{sc} = -0.1353 \text{ p.u.} \quad V_{sc} = 0.9528 \text{ p.u.}$$

with

	k_1(p.u.)	α_1(rad)	k_2(p.u.)	α_2(rad)	k_1V_{dc}(p.u.)
P − 1	0.9626	0.0017	0.0024	−2.8803	0.9533
P − 2	0.1287	0.0990	0.0086	1.6396	0.9891

The negative sign in P_{sc} and Q_{sc} indicates that the power flow is from bus 2 to bus 1. Both P-1 and P-2 satisfy the same injection model constraints but with significantly different results. The P-1 system eigenvalues all lie in the left half plane, whereas a pair of P-2 system eigenvalues have migrated to the right half plane. To see the difference in the effect of the operating points, consider a three-phase ground fault on bus 30 (of the IEEE 118-bus test system) cleared after 0.12 s (see Fig. 4). For the system initialized at P-1, the oscillations remain bounded,

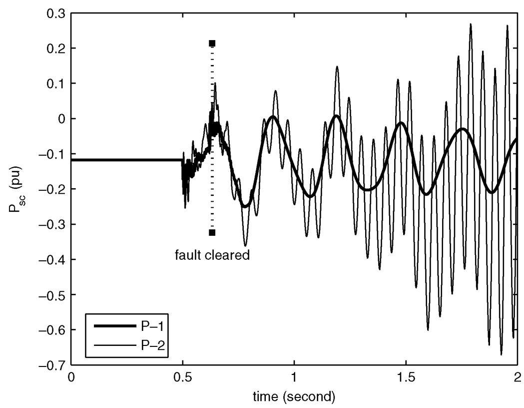

Fig. 4. Dynamic response of UPFC series active power.

whereas the system initialized at P-2 results in nonlinear undamped oscillations.

IV. Summary and Conclusions

This letter is intended as a cautionary note for the use of the power injection model. While the power injection model is a useful simplification, it does not represent losses and may therefore lead to inaccurate estimates of the stability and dynamic behavior of the full system.

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